# Determining Feasibility of Small Distance Measurement Utilizing a Mach-Zehnder Interferometer with Single-Frequency Lasers

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Since its inception by Michelson and Morley, interferometry has been an important investigative technique, leading to advancements such as detecting gravitational waves from colliding neutron stars[2] and producing model-independent images of astronomical targets at high resolutions.[3] We design a procedure for distance measurement using a Mach-Zehnder interferometer, and evaluate its viability as a cost-efficient alternative to leading optical distance measurement techniques.

# INTRODUCTION

The study of interferometry concerns making precise conclusions about a surface or path through interference of electromagnetic waves. Optics is part of every physicist's foundation of knowledge, but due to the high cost of lab equipment, most students do not get the opportunity to apply these concepts and conduct meaningful experiments. We are interested in pursuing this topic to explore what educational value can be gained from lowcost interferometry, specifically distance measurement.

#### **Interferometry Background**

Given a coherent, single-frequency laser source, we can use beamsplitters and mirrors so the path of a coherence laser beam diverges, then reconverges. With proper alignment, an interferogram is visible where the beam exits the setup. From either the intensity of the interferogram and the interferogram itself, which looks like a target, we can determine the phase difference between the two arms of the interferometer.

#### Homodyne Detection and Coherence Length

In contrast to existing optical distance measurement setups, which employ heterodyne detection (e.g. frequency combing with an acousto-optical modulator), we designed a procedure with a single-frequency laser source (homodyne detection).[5] The most important criteria for the two lasers we selected were 1) budget feasibility and 2) coherence beyond the optical path length difference in our tabletop setup.

We chose a 633 nm helium-neon source for its availability, cost efficiency, stability, low power output (1 mW), and coherence length; this is between 20 cm and 100 m, depending on whether the HeNe laser is multimodal or single-modal (pulsed).[4] Due to budget constraints, our second laser was a 532 nm diode laser with a coherence length too short for us to observe any interference.

# OUR MACH-ZEHNDER INTERFEROMETER

With distance measurement as the end goal, we modified the classic Mach-Zehnder interferometer setup by rotating the final beamsplitter  $90^{\circ}$ . The upper beam path thus travels to and from an additional mirror.

Equipment on a \$400 Budget Constraint	
Cost	Item
N/A	Optics table or board
N/A	CMOS camera
N/A	optics mirrors 5x
N/A	$1 \mathrm{~cm}$ polarizing beamsplitters $2 \mathrm{x}$
N/A	beam splitter posts $3x$
\$220	1 in. non-polarizing beamsplitter
N/A	633  nm (red) HeNe laser
\$15	532 nm (green) diode laser

# **Our Setup**

There are crucial elements to our setup to accommodate our equipment and budget constraints, and we were able to learn from our many revisions to the setup.

First, attenuation of the light is necessary for the camera to capture the interference patterns without getting damaged. In this experiment, we use wave-plates and an extra beamsplitter put in front of the light-source to attenuate the light.

Second, beamsplitter C has to be a non-polarizing beamsplitter. A polarizing beamsplitter splits light by its polarization components and is unable to recombine the light in our setup; this is because the upper arm beam requires a certain polarization to pass through beamsplitter C, while requiring the opposite polarization to be then reflected from beamsplitter C towards the camera.

Finally, coordinating the initial height and direction of the laser is crucial for proper alignment. Precision of the alignment of the beams is key to perfectly recombine the beams, and to obtain the wanted interference patterns.



FIG. 1: Via the first two mirrors, the laser beam is aligned parallel to the optics table and the holes in the optics table. It diverges and reconverges following the classic Mach-Zehnder setup until beamsplitter C, where the upper path P' transmits through, then is reflected by, beamsplitter C. The waveplates attenuate the beam and allow for adjustment of the relative path intensities.

Without the coordination of the initial beam of light, aligning the split beams is impossible without the help of additional optics equipments.

After we realized these needs, we were finally able to come to a working set that attenuates the light and recombines the beams successfully.

#### Alignment Procedure

Precise alignment is fundamental in order for the split beams to reconverge. Taking meticulous control over the setup is necessary and we did through these steps:

- 1. Adjust the height and the direction of the initial beam of light using two mirrors so that the light travels parallel to the optics table and the holes in the optics table at an appropriate height.
- 2. Attenuate the initial beam of light by using waveplates and a beamsplitter.
- 3. Adjust the angle of mirrors so the light beams travel in 90 degree angles at a constant height.
- 4. Position and angle the last beamsplitter that reconverges the beams precisely.
- 5. Adjust the mirrors in tiny increments to find circular diffraction center.

## Design Decisions

Justifying the key choices regarding our methodology:

- Mach-Zehnder over Michelson interferometry because we could easily extend one of the arms
- Two lasers of differing frequencies to resolve phase periodicity (refer to the Calculations section).
  - Second harmonic generation crystals are not viable, as there must be unique phase data.
- Beamsplitter C must be non-polarizing, as opposed to polarizing, for the upper path P' to be both transmitted through and reflected it.
  - A glass slide, while more affordable, did not suffice due to its thickness causing interference
- Aligned the lasers together with beamsplitter A. Would have used optical fibers if we had more time
- Analyzed the interferogram itself instead of its intensity because we had a camera
- Removed a beam detour shortening the path length difference as the green laser did not interfere
  - Instead used the two mirrors to align the laser with the holes and height of the optics table

#### Calculations

To calculate the optical path length difference from the phase difference, we must consider the phase changes to the beam along both the upper and lower paths. The lower path is reflected by mirror C, then goes through beamsplitter C. The upper path is reflected by mirror D, goes through beamsplitter C, is reflected by mirror E, then gets reflected by beamsplitter C.

Let the laser sources have wavelengths  $\lambda_1$  and  $\lambda_2$ , respectively. The phase change for laser 1 along path P (lower) is

$$\phi_{1,P} = \frac{2\pi}{\lambda_1} |P| + C_{1,P} \tag{1}$$

and the phase change of laser 1 along path P' is

$$\phi_{1,P'} = \frac{2\pi}{\lambda_1} |P'| + C_{1,P'} \tag{2}$$

where the C's are constants. The phase difference is thus

$$\phi_1 = \phi_{1,P'} - \phi_{1,P} = \frac{2\pi}{\lambda_1}(2d) + C_1 \tag{3}$$

where C is the due to differing optical path lengths through the beamsplitters. Similarly, the phase difference between lasers 1 and 2 along path P' (upper) is

$$\phi_2 = \phi_{2,P'} - \phi_{2,P} = \frac{2\pi}{\lambda_2}(2d) + C_2 \tag{4}$$

For two values of the distance between mirror E and beamsplitter C,  $d = d_1$  and  $d = d_2$ , we can, in theory, find  $\Delta d = d_2 - d_1$ . In practice, having only one laser with sufficient coherence length, we could not observe the data we had hoped for, much less conduct error analysis.

In summary, by finding two values each of  $\phi_1 \pmod{\pi}$ and  $\phi_2 \pmod{\pi}$ , we reduced the distance measurement problem to finding two integers  $m_1$  and  $m_2$  such that

$$\frac{\lambda_1}{4\pi}(\Delta\phi_1 + m_1\pi) = \frac{\lambda_2}{4\pi}(\Delta\phi_2 + m_2\pi) = \Delta d \qquad (6)$$

## RESULTS

Although with only one coherent laser source, we were unable to measure the distance as we initially intended, we still observed an interference pattern from the red laser on over a dozen occasions. We verified that this was a true interference pattern by blocking one arm at a time, as in Figure 2.



FIG. 2: Left: interference when both arms are unblocked Center: Upper path only; Right: Lower path only

Over  $90^{\circ}$  of our time on this project went towards aligning the beamsplitter to find the center of the airy disk. This was extremely difficult because we originally purchased a 5 mm non-polarizing beamsplitter. As a result, it was too difficult to find the center of the interferogram because in our search, the laser would exit the beamsplitter.



FIG. 3: When we purchased a 1 in. (25.4 mm) beamsplitter, the alignment process became much easier, and we succeed in finding the center. It was too zoomed in to see a proper airy disk with our CMOS camera, so the two images here are the frames of a video using Photoshop's layer difference capability, and a cell phone camera image.

# Mathematical Analysis of the Interferograms

Due to geometrical symmetry, two symmetric spectral sidelobes carry the same information about  $\Delta\phi$ . By using appropriate bandpass filters, we can choose one of the two, and we can apply an inverse Fourier Transformation to obtain values of  $\Delta\phi$ . For more details, refer to [6]. We found MATLAB code online to extract the phase from an interferogram, but it did not work on our interferograms in Figure 3 because they were not of sufficient quality.

## CONCLUSIONS

In this paper, we have outlined a clear procedure we designed to perform distance measurement with a modified Mach-Zehnder interferometer. Despite our equipment not being sufficient to observe an interferogram from a second laser source, the laser source we did observe an airy disk for serves to validate our proposed methodology. We have concluded that homodyne detection distance measurement is theoretically sound, as well as found airy disk patterns as proof of concept that this experiment works and is suitable for college-aged physicists. However, questions remain that require more money, time, and data, including:

- Given quality airy disks, how reliable is code to extract the phase from an interferogram?
- Considering the phase periodicity, how much error can be expected in the final distance calculation?

Even though we could not feasibly obtain the quantitative data we had wanted, there are insights into setting up this experiment that we can take away from the project that would help with constructing this experiment in future. We believe that the project has potential and can be achieved with more time and resources. Throughout the time that we spent on the experiment we have faced many of challenges, and if we were to start it all over but with our earned experience, we would have been able to achieve our goal in time.

If we had a larger budget, we would be able purchase a second laser with a longer coherence length to be coupled to the HeNe laser with an optical fiber instead of alignment through a beamsplitter. We could also purchase optical rails to expedite and/or automate the alignment process.

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